Permittivity Measurements Using a Short Open-Ended Coaxial Line Probe

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Abstract—**Improvements for permittivity measurements with short, open-ended coaxial line probes are achieved by using a more accurate formulation and by adding a finite conductor flange at the aperture of the probe. A conductor flange with a diameter of about 10 times the outer diameter of the coaxial line greatly improves the performance of the coaxial line probe. For a probe without a flange, a three-term admittance formula with calculated coefficients gives good measured results as compared to standards. However, for a probe with flange, a two-term admittance formula with a calibration coefficient gives better measured results.**

I. Introduction

There are many ways to measure the complex permittivity of a particular material, using methods employing free space, resonator, transmission, and/or reflection measurements. The reflection method, in general, is the most promising approach, especially when used with an open-ended coaxial line probe as shown in Fig. 1(a). A quasi-static analysis for this probe with an infinite ground plane at the aperture has been published [1], based on a three-term theoretical formula for the calculation of the probe aperture admittance from the complex permittivity of the medium filled half-space. Recently this probe was investigated by Staebell *et al.* [2], where a two-term approximation admittance formula with calibration coefficient was employed. This probe is based on an open-ended co-ax cable without any conducting flange to simulate the infinite ground plane model and only two terms of a three-term formula are used for permittivity calculation. Therefore, it was felt that two things needed further investigation. The first is a study of the effect of the use of all three terms in the quasi static analysis formulation for the open-ended co-ax probes, and the second is the determination of the effect of a conductor flange at the probe aperture. The Hewlett-Packard Company has recently begun marketing a model HP8507A flanged probe for determining complex permittivity of solids and liquids for use with network analyzers which further emphasizes this interest.

To investigate these effects, an open-ended coaxial line probe was constructed as shown in Fig. 1(a), and a conductor flange with a diameter of about 10 times the outer diameter of the coaxial line was constructed as shown in Fig. 1(b). Several liquids with known permittivity were tested using a Hewlett-Packard 8510 network analyzer for measurement.

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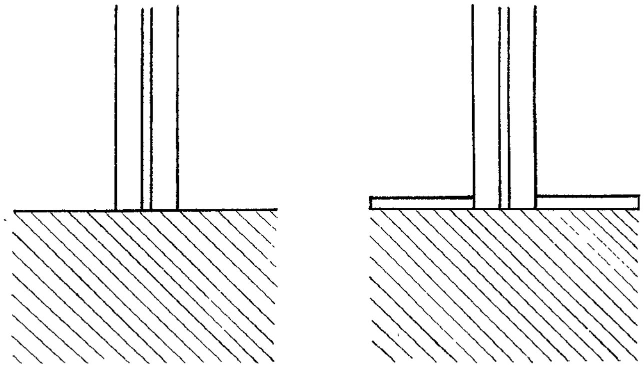

(a) (b)

Fig. 1. Open-ended coax probe structures. (a) without flange and (b) with flange.

Data from both the two-term approximation formula with calibration coefficient and the three-term approximation formula with calculation coefficients were used to interpret the measured data for complex permittivity as compared to standard values [3]. The basic theory of the probe permittivity measurement is discussed in the next section, and the calculations and comparison of results follow. Further background information on related probe measurements can be found in [4] and [5].

II. Quasi-Static Analysis

A coaxial line open to an infinite ground plane where the half-space is filled with an isotropic, homogenous medium with a complex permittivity ϵ is investigated for an infinite ground plane model similar to models shown in Fig. 1. Only the dominant mode is assumed to propagate inside the coaxial line. At the aperture, the incident wave causes radiation into the half-space and, consequently, causes a multimode reflected wave to exist inside the coaxial line. From image theory and the existing boundary condition, an expression for the aperture admittance has been derived as [1]

$$Y_L = \frac{j2\omega I_1}{[\ln(b/a)]^2}\epsilon - \frac{j\omega^3\mu_0 I_2}{[\ln(b/a)]^2}\epsilon^2 + \frac{\pi\omega^4\mu_0^{3/2}}{12}\left[\frac{b^2-a^2}{\ln(b/a)}\right]^2\epsilon^{5/2}, \quad (1)$$

where ω is the operating angular frequency and a and b are the inner and outer radii of the coaxial line, respectively. I_1 and I_2 are integration constants dependent only on the geometry of the aperture. This formula is referred to as a three-term approximation with calculation coefficients [1], and it has been used to obtain approximate analyzes [2] for the widely used open-ended co-ax probe without ground plane or flange [6].

III. Bilinear Transform

When using a network analyzer to measure reflection coefficient of the probe, all the measurements are referred to the interface between the network analyzer and the probe. However in calculating the permittivity, the admittance at interface of the probe and the sample is needed. Therefore, a bilinear transform is used to account for the effect of the probe and the system imperfections [2]. The bilinear transformation used in this study is:

$$\frac{Y_s - Y_1}{Y_s - Y_2} \cdot \frac{Y_3 - Y_2}{Y_1 - Y_3} = \frac{\Delta_{s1}\Delta_{32}}{\Delta_{s2}\Delta_{13}},$$

where

$$\Delta_{ij} = \rho_i - \rho_j,$$

and ρ_i is the measured reflection coefficient. Y_1, Y_2, and Y_3 are the admittances of three standard materials when observed at the end of the probe. After calibration with three standards, this formula can be used to transform the measured reflection coefficients into the effective admittance observed at the aperture of the probe for a circuit theory model.

IV. Simplified Two Term Approximation Formula

In the quasi-static formula (1), the third term with $\epsilon^{5/2}$ causes some difficulty in solving for the permittivity because of the added complexity of computation. Staebell, *et al.* used only a two-term approximation formula,

$$Y_L = \epsilon_r + \zeta\epsilon_r^2, \tag{2}$$

where ϵ_r is the relative complex permittivity of the medium and ζ is a calibration coefficient that can be calculated with the use of a standard material. This equation is the two-term formula used to determine permittivity [2].

V. Experimental Investigation

The following permittivity measurements are obtained by using a HP 8510B network analyzer for a frequency range from 500 MHz to 5.0 GHz with data averaging for 8 samples.

The probe configuration investigated in this approach are:

- No. 1: the probe without flange and admittance formula (2),
- No. 2: the probe without flange and admittance formula (1),
- No. 3: the probe with flange and admittance formula (2),
- No. 4: the probe with flange and admittance formula (1).

The probe without flange was inserted 3 centimeters into the liquid for the reported measurements. When the three-term admittance formula (1) is used, water, ethyl acetate, and an open circuit are used as the three required standards. When the two-term admittance formula (2) is used, water, and open-circuit, a short circuit, and ethyl acetate are used as the four required standards. Acetone, methanol, and tap water are chosen as testing liquids. Then, the measured data for permittivities are fitted to the Cole–Davidson equation for representation of the smoothed permittivity data [3],

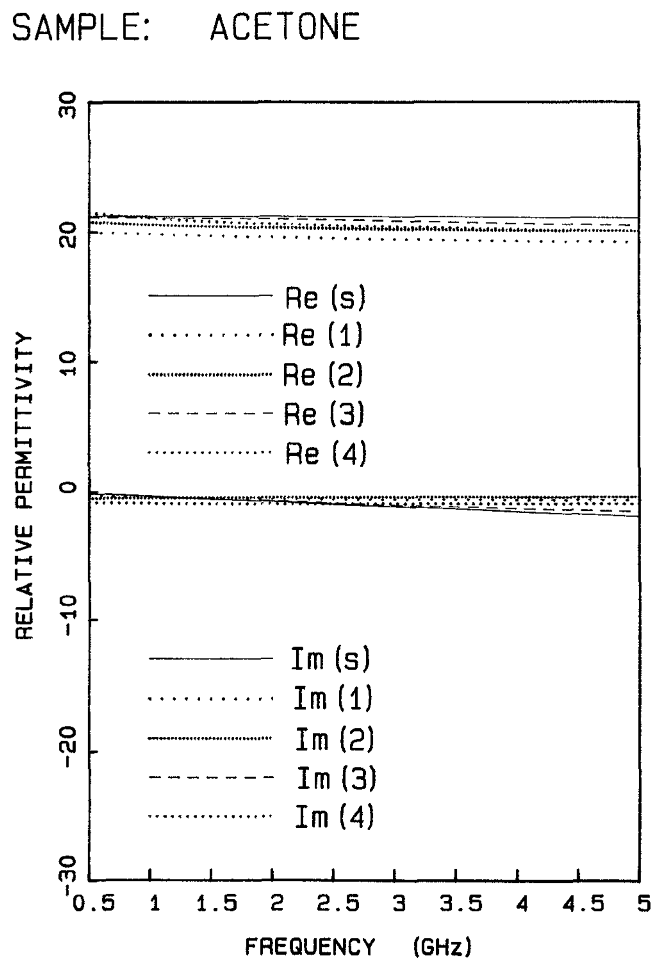

Fig. 2. Measured permittivity for Acetone at 20°C.

$$\epsilon_r = A_1 + \frac{A_2}{(1 + j\omega A_3)^{A_4}}, \tag{3}$$

where A_1, A_2, A_3, and A_4 are real optimized coefficients determined by a least square curve fitting method.

The measured relative complex permittivities of acetone and methanol as fitted to (3) are plotted in Fig. 2 and Fig. 3, respectively. The numbered curves represent different probes and different admittance formulas. The standard curves from [3] are also plotted for comparison. The measured results for tap water are even closer to the standard values and give similar results, but are omitted for brevity.

VI. Conclusion

A comparison of the measured permittivity plotted for both acetone and methanol can be made from the four curves (No. 1, No. 2, No. 3 and No. 4) included in each figure. The differences in the curves show that a short probe with a finite conductor flange has a large effect on the performance of this measurement, because it enforces the field boundary condi-

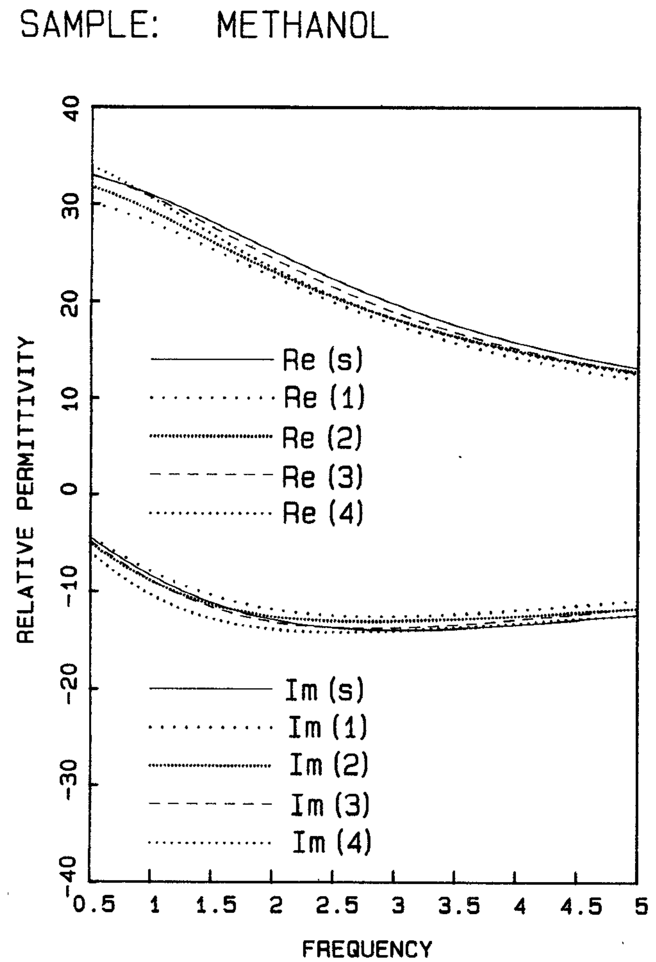

Fig. 3. Measured permittivity for Methanol at 20°C.

tions inside the testing liquid in a manner similar to the assumed infinite ground plane, particularly in the local region of the aperture. A comparison of curves No. 1 and No. 2 in both Fig. 2 and Fig. 3 indicates that when the probe has no flange, the three-term formula of (1) works better, and from curves No. 3 and No. 4, it is obvious that when the probe has a flange, the two-term formula of (2) is better for the measured results for both acetone and methanol.

Thus, this investigation indicates that for an open-ended coax line probe, the three-term formula does indeed give more accurate results than the two-term formula for this type probe. For the flanged probe, these results show that the two-term formula provides the more accurate measurement; however, the three-term formula gives results that are almost as good as the two-term formula for the flanged probes. Thus, it is further concluded from these data that the use of flanged probes result in more accurate measurements because the flange model more closely represents the model used to derive the formula of (1).

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